To multiply positive numbers y and z:

1. Convert y and z to their logarithms, $\log y$ and $\log z$.
2. Let l be the sum of $\log y$ and $\log z$.
3. Let x be the number such that $\log x = l$.
4. Terminate with answer x.

Algorithm 1.5 Multiplication using logarithms.

Our story now moves to Europe. The Scots scholar John Napier (1550–1617), observing that computations involving multiplication, division, powers, and roots were slow and error-prone, invented logarithms. The logarithm of a number y is the number of times that 10 must be multiplied by itself to give y; thus $\log_{10}10 = 1$, $\log_{10}100 = 2$ (since $10^2 = 100$), $\log_{10}1000 = 3$ (since $10^3 = 1000$), and so on. Intermediate numbers have fractional logarithms; thus $\log_{10}52 = 1.716$, accurate to 3 decimal places.

EXAMPLE 1.4 *Multiplication using logarithms*

The basic law of logarithms is:

$$\log(y \times z) = \log y + \log z$$

This leads directly to a multiplication algorithm: see Algorithm 1.5.

This algorithm is faster and less error-prone than the long multiplication algorithm. A number can be quickly converted to a logarithm (step 1) or from a logarithm (step 3) by looking up a table of logarithms.

Logarithms (and their mechanical equivalents, slide rules) quickly became essential tools for scientists, engineers, astronomers, and everyone else who had to do complex calculations. They continued to be used routinely until they were superseded by electronic calculators and computers in the 1960s.

The English mathematician and scientist, Isaac Newton (1643–1727), invented the differential calculus. This gave us algorithms for differentiation (determining the slope of a given curve) and integration (determining the area under a given curve). Differential calculus became a keystone of mathematics and physics, and has found a vast array of applications in engineering, weather forecasting, economics, and so on.

EXAMPLE 1.5 *Computing a square root*

Computing the square root of a positive number a is equivalent to finding the side of a square whose area is a. Algorithms for computing square roots approximately have been known since ancient times, but Newton's algorithm is preferred in modern computation.

Newton's algorithm is a by-product of differential calculus, but can be understood